EXPONENTIAL FUNCTION

An exponential function, where $a > 0$ and $a \neq 1$, is a function of the form

$$f(x) = a^x$$

$f(x) = 3x + 4$ $\qquad\qquad$ $f(x) = x^2 - 4$ $\qquad\qquad$ $f(x) = 2^x$

$$x^3 + 7$$

$$x^{1/2} - 3$$

On the same coordinate system graph $f(x) = 2^x$ and $g(x) = 3^x$.
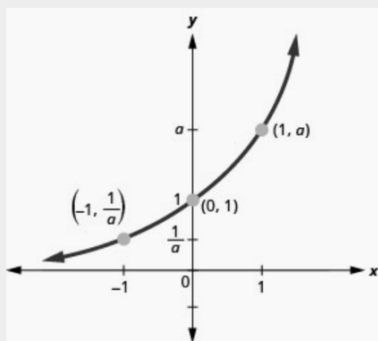
$D: (-\infty, \infty)$

$R: (0, \infty)$

$2^0 = 1 \qquad 3^0 = 1$

$2^x = 0$

$2^{-1} = \frac{1}{2}$

PROPERTIES OF THE GRAPH OF $f(x) = a^x$ **WHEN** $a > 1$

Domain	$(-\infty, \infty)$
Range	$(0, \infty)$
x-intercept	None
y-intercept	$(0, 1)$
Contains	$(1, a)$, $\left(-1, \frac{1}{a}\right)$
Asymptote	x-axis, the line $y = 0$

2^x $(1, 2)$ $\left(-1, \frac{1}{2}\right)$

3^x $(1, 3)$ $\left(-1, \frac{1}{3}\right)$

5^x $(1, 5)$ $\left(-1, \frac{1}{5}\right)$

On the same coordinate system, graph $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \left(\frac{1}{3}\right)^x$.

PROPERTIES OF THE GRAPH OF $f(x) = a^x$

when $a > 1$		when $0 < a < 1$	
Domain	$(-\infty, \infty)$	Domain	$(-\infty, \infty)$
Range	$(0, \infty)$	Range	$(0, \infty)$
x-intercept	none	x-intercept	none
y-intercept	$(0, 1)$	y-intercept	$(0, 1)$
Contains	$(1, a)$, $\left(-1, \frac{1}{a}\right)$	Contains	$(1, a)$, $\left(-1, \frac{1}{a}\right)$
Asymptote	x-axis, the line $y = 0$	Asymptote	x-axis, the line $y = 0$
Basic shape	increasing	Basic shape	decreasing
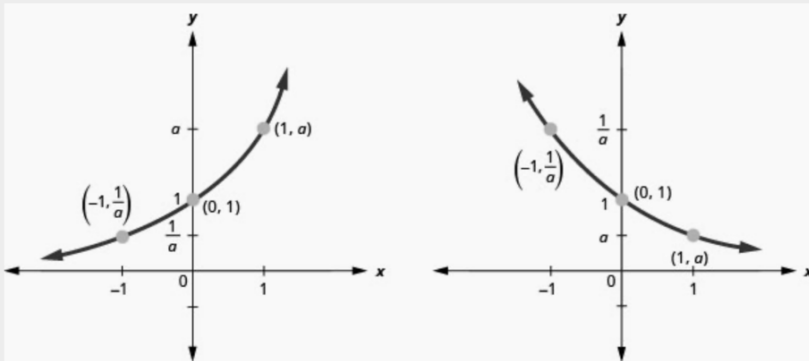
$\frac{1}{2}^{x}$ $\left(1, \frac{1}{2}\right)$
$\left(-1, 2\right)$

$\frac{1}{3}^{x} = \left(1, \frac{1}{3}\right)$
$\left(-1, 3\right)$

On the same coordinate system graph $f(x) = 2^x$ and $g(x) = 2^{x+1}$.

x	y
0	1
1	2
2	4
3	8

x	2^{x+1}
0	2
1	4
2	8
3	16

$\sqrt{x}$ $\sqrt{x+1}$

$\hookrightarrow$ shift
Left 1

On the same coordinate system, graph: $f(x) = 2^x$ and $g(x) = 2^{x-1}$.

Shift Right 1

On the same coordinate system, graph: $f(x) = 3^x$ and $g(x) = 3^{x+1}$.

On the same coordinate system, graph: $f(x) = 3^x$ and $g(x) = 3^x + 2$.

On the same coordinate system graph $f(x) = 3^x$ and $g(x) = 3^x - 2$.

Horizontal Shift

↓

$$f(x) = a^{x-h} + k$$

↓

Vertical Shift

NATURAL BASE e

The number e is defined as the value of $\left(1 + \frac{1}{n}\right)^n$, as n increases without bound. We say, as n approaches infinity,

$$e \approx 2.718281827...$$

NATURAL EXPONENTIAL FUNCTION

The natural exponential function is an exponential function whose base is e

$$f(x) = e^x$$

The domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.

$(1, e)$

$\left(-1, \dfrac{1}{e}\right)$

How to Solve an Exponential Equation

Solve: $3^{2x-5} = 27.$

$$27 = 3^3$$

$$3^{2x-5} = 3^3$$

$$2x - 5 = 3$$

$$2x = 8$$

$$x = 4$$

Solve: $3^{3x-2} = 81$.

$$81 = 3^4$$

$$3^{3x-2} = 3^4$$

$$3x - 2 = 4$$

$$3x = 6$$

$$x = 2$$

Solve: $7^{x-3} = 7$.

$$2^{x+3} = 5$$

$$7^{x-3} = 7^1$$

$$x - 3 = 1$$

$$x = 4$$

How to Solve an Exponential Equation

Step 1. Write both sides of the equation with the same base, if possible.

Step 2. Write a new equation by setting the exponents equal.

Step 3. Solve the equation.

Step 4. Check the solution.

Solve $\frac{e^{x^2}}{e^3} = e^{2x}$.

$$e^{x^2-3} = e^{2x}$$

$$x^2 - 3 = 2x$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$x = 3$ $x = -1$

Solve: $\dfrac{e^{x^2}}{e^x} = e^6$.

$$e^{x^2 - x} = e^6$$

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$\boxed{x = 3} \quad \boxed{x = -2}$$

COMPOUND INTEREST

For a principal, P, invested at an interest rate, r, for t years, the new balance, A, is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad \text{when compounded } n \text{ times a year.}$$

$$A = Pe^{rt} \qquad \text{when compounded continuously.}$$

$\hookrightarrow \# e$

$P = \text{Principal} \rightarrow \text{Start with}$

$r = \% \text{ rate (decimal)}$

$t = \text{time years}$

Compound

Yearly $\quad n = 1$

Quarterly $\quad n = 4$

Monthly $\quad n = 12$

Weekly $\quad n = 52$

EXAMPLE 10.16

A total of $10,000 was invested in a college fund for a new grandchild. If the interest rate is 5%, much will be in the account in 18 years by each method of compounding?

ⓐ compound quarterly

ⓑ compound monthly

ⓒ compound continuously

a) $n = 4$
$P = 10,000$
$r = .05$
$t = 18$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 10,000\left(1 + \frac{.05}{4}\right)^{(4)(18)}$$

$$= \$24,459.20$$

b) $$A = 10,000\left(1 + \frac{.05}{12}\right)^{(12)(18)}$$

$$= 24550.08$$

c) $= A = Pe^{rt}$

$$10,000 \, e^{(.05)(18)}$$

$$= \$24,596.03$$

Angela invested $15,000 in a savings account. If the interest rate is 4%, how much will be in the account in 10 years by each method of compounding?

ⓐ compound quarterly

ⓑ compound monthly

ⓒ compound continuously